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CORONA PRODUCT OF PRODUCT FUZZY GRAPHS

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Abstract: In this article, a new operation on product fuzzy graphs (PFGs) is provide; namely corona product. We give sufficient conditions for the corona product of two PFGs to be complete. We also study the unbiased notion of the class of PFGs and necessary and sufficient conditions for the corona product to be unbiased are given.

Keywords and Phrases: PFG, complete PFG, unbiased PFG, corona product.

2020 Mathematics Subject Classification: 05C72.

1. Background

Graph theory has many applications in mathematics and economics. Since most problems of graphs are undetermind, it is necessary to handel these facets via the method of fuzzy logic. Fuzzy relations were introduced by Zadeh [22] in 1965. Rosenfeld [19] in 1975, introduced fuzzy graphs (simply, FG) and some ideas that are generalizations of those of graph's. Now adays, this theory is having more and more applications in which the informatiion level immanent in the system differ with various levels of accuracy. Fuzzy models are convenient as they reduce differences between long-established numerical models of expert systems and symbolic models. Peng and Mordeson [14] defined the conceptualization of FG's complement and conscious FG's operations. In [21], improved complement's definition in order to guarantee the original FG is isomorphic to complement of the complement, which concur with the case of crisp graphs. In addition, self-complementary

FGs properties and the complement under FG's join, union and composition (introduced in [14]) were explored. Al-Hawary [2] introduced the concept of balanced in the class of FGs and Al-Hawary and others have deeply explored this ides for many types of FGs. For more on the foregoing concepts and those coming after ones, one can see [1, 2, 3, 4, 5, 6, 7, 8, 9, 14, 16, 17, 18, 21].

A mapping $ţ$:$\H{U}\rightarrow [0,1]$ is a fuzzy subset of a nonempty set $\H{U}$ and a fuzzy subset of $\H{U}\times\H{U}$ is called a fuzzy relation ς on $ţ$. We assume that $ţ$ is reflexive, $\H{U}$ is finite and ς is symmetric.

Definition 1. [19] *A fuzzy graph (simply, FG), with $\H{U}$ as the underlying set, is a pair $Ģ$: $(ţ,\varsigma)$ where $ţ$:$\H{U}\rightarrow [0,1]$ is a fuzzy subset and ς :$\H{U}\times\H{U}\rightarrow [0,1]$ is a fuzzy relation on $ţ$ such that $\varsigma(c,s) \leq ţ(c)\wedge ţ(s)$ for all $c,s \in\H{U}$, where $\wedge$ stands for minimum. The crisp graph of $Ģ$ is denoted by $Ģ^* : (ţ^*,\varsigma^*)$ where $ţ^* = \sup c(ţ) = \{c \in\H{U}:ţ(c) > 0\}$ and $\varsigma^* = \sup c(\varsigma) = \{(c,s) \in\H{U}\times\H{U}: \varsigma(c,s) > 0\}.H = (ţ',\varsigma')$ is a fuzzy subgraph of $Ģ$ if there exists $c \in\H{U}$ such that $ţ' : c \rightarrow [0,1]$ is a fuzzy subset and $\varsigma' : c\times c \rightarrow [0,1]$ is a fuzzy relation on $ţ'$ such that $\varsigma(c,s) \leq ţ(c)\wedge ţ(s)$ for all $c,s \in c$.*

Definition 2. [19] *Two FGs $Ģ_1 : (ţ_1,\varsigma_1)$ are isomorphic if there exists a bijection $h : \H{U}_1 \rightarrow \H{U}_2$ such that $ţ_1(c) = ţ_2(h(c))$ for all $c \in \H{U}_1$ and $\varsigma_1(c,s) = \varsigma_2(h(c),h(s))$ for all $(c,s) \in \breve{E}_1$. We then write $Ģ_1 \simeq Ģ_2$ and h is called an isomorphism.*

Using the operation of product instead of minimum, Ramaswamy and Poornima in [20] established PFGs.

Definition 3. [20] *Let $Ģ^* : (\H{U},\breve{E})$ be a graph, $ţ$ be a fuzzy subset of $\H{U}$ and ς be a fuzzy subset of $\H{U}\times\H{U}$.We call $Ģ$: $(ţ,\varsigma)$ a product fuzzy graph (simply, PFG) if $\varsigma(c,s) \leq ţ(c)ţ(s)$ for all $c,s \in\H{U}$.*

The following result is immediate:

Lemma 1. *Every PFG is a FG, but the converse need not be true.*

Definition 4. [20] *A PFG $Ģ$: $(ţ,\varsigma)$ is called complete if $\varsigma(c,s) = ţ(c)ţ(s)$ for all $c,s \in\H{U}$.*

Definition 5. [20] *A PFG $Ģ$: $(ţ,\varsigma)$ is called strong if $\varsigma(c,s) = ţ(c)ţ(s)$ for all $(c,s) \in\breve{E}$.*

Definition 6. [20] *The complement of a PFG $Ģ$: $(ţ,\varsigma)$ is $Ģ^c : (ţ^c,\varsigma^c)$ where $ţ^c = ţ$ and*

$$\begin{aligned}\varsigma^c(c,s) &= ţ^c(c)ţ^c(s) - \varsigma(c,s)\\ &= ţ(c)ţ(s) - \varsigma(c,s).\end{aligned}$$

Lemma 2. [7] *If $Ģ$: $(ţ, \varsigma)$ is a self-complementary PFG, then*

$$\sum_{(c,s)\in \breve{E}} \varsigma(c,s) = \frac{1}{2} \sum_{(c,s)\in \breve{E}} ţ(c)ţ(s).$$

Lemma 3. [7] *Let $Ģ$: $(ţ, \varsigma)$ be a PFG such that $\varsigma(c,s) = \frac{1}{2}ţ(c)ţ(s)$ for all $c, s \in \H{U}$. Then $Ģ$ is self-complementary.*

Several types of products of two FGs were explored. The notion of corona product of FGs was introduced and studied in [11] where the regularity property for this product was the main idea. In Section 2 of this paper, we launch the conception of corona product of PFGs. We prove that the corona product of two PFGs to be is complete if and only if both factors are complete PFG. Section 3 is devoted to give necessary and sufficient conditions for the corona product of two unbiased PFGs to be unbiased.

2. Corona Product of Product Fuzzy Graphs

In [13], the corona product $G_1 \circ G_2$ of two classical graphs G_1 and G_2 is the graph consisting of one copy of G_1 of order n and n copies of G_2 where the i^{th} vertex of G_1 is joined by an edge to the i^{th} copy of G_2. It was proved that corona product is neither associative nor commutative. In [11], corona product of two fuzzy graphs where introduced and degrees of vertices was the main goal to study. We begin this section by defining the rooted product of PFGs.

Definition 7. *The corona product of two disjoint PFGs $Ģ_1 : (ţ_1, \varsigma_1)$ of order n and $Ģ_2 : (ţ_2, \varsigma_2)$ is defined to be the FG $Ģ_1 \propto Ģ_2 : (ţ_1 \propto ţ_2, \varsigma_1 \propto \varsigma_2)$ on the vertex set $\H{U}_1$ union n copies of $\H{U}_2$, where*

$$(ţ_1 \propto ţ_2)(\H{u}) = \begin{cases} ţ_1(\H{u}) & \H{u} \in \H{U}_1 \\ ţ_2(\H{u}) & \H{u} \in \H{U}_2 \end{cases} \text{ and}$$

$$(\varsigma_1 \propto \varsigma_2)(\H{u}_1, \H{u}_2) = \begin{cases} \varsigma_1(\H{u}_1\H{u}_2) & \H{u}_1\H{u}_2 \in \breve{E}_1 \\ \varsigma_2(\H{u}_1\H{u}_2) & \H{u}_1\H{u}_2 \in \breve{E}_2 \\ ţ_1(\H{u})ţ_2(\H{u}_2) & \H{u}_1\H{u}_2 \in \breve{E} \end{cases}$$

where $\breve{E}$ is the set of edges joining each vertex of $Ģ_1$ to the vertices of the n copies of $Ģ_2$.

It is clear that the above definition is well-defined. That is, the corona product of two PFGs is a PFG.

Theorem 1. *If $\c{G}_1 : (ţ_1, \varsigma_1)$ and $\c{G}_2 : (ţ_2, \varsigma_2)$ are strong PFGs, then $\c{G}_1 \propto \c{G}_2$ is a strong PFG.*

Proof. Let $\c{G}_1 : (ţ_1, \varsigma_1)$ and $\c{G}_2 : (ţ_2, \varsigma_2)$ be two strong PFGs.

Case 1: If $\H{u}_1\H{u}_2 \in \breve{E}_1$, then as $\c{G}_1$ strong,

$$\begin{aligned}(\varsigma_1 \propto \varsigma_2)(\H{u}_1, \H{u}_2) &= \varsigma_1(\H{u}_1\H{u}_2)\\ &= ţ_1(\H{u}_1)ţ_1(\H{u}_2)\\ &= (ţ_1 \propto ţ_2)(\H{u}_1)(ţ_1 \propto ţ_2)(\H{u}_2).\end{aligned}$$

Case 2: If $\H{u}_1\H{u}_2 \in \breve{E}_2$, is similar to case 1.

Case 3: If $\H{u}_1\H{u}_2 \in \breve{E}$, then by definition

$$\begin{aligned}(\varsigma_1 \propto \varsigma_2)(\H{u}_1, \H{u}_2) &= ţ_1(\H{u})ţ_2(\H{u}_2)\\ &= (ţ_1 \propto ţ_2)(\H{u}_1)(ţ_1 \propto ţ_2)(\H{u}_2).\end{aligned}$$

Case 3: If $\H{u}_1\H{u}_2 \in \breve{E}$, then by definition

$$\begin{aligned}(\varsigma_1 \propto \varsigma_2)(\H{u}_1, \H{u}_2) &= ţ_1(\H{u})ţ_2(\H{u}_2)\\ &= (ţ_1 \propto ţ_2)(\H{u}_1)(ţ_1 \propto ţ_2)(\H{u}_2).\end{aligned}$$

Thus, $\c{G}_1 \propto \c{G}_2$ is a strong PFG.

Corollary 1. *If $\c{G}_1 : (ţ_1, \varsigma_1)$ and $\c{G}_2 : (ţ_2, \varsigma_2)$ are fuzzy complete PFGs, then $\c{G}_1 \propto \c{G}_2$ is a complete PFG.*

Next, we show that if the corona product of two PFGs is complete, then at least one of the two PFGs must be complete.

Theorem 2. *If $\c{G}_1 : (ţ_1, \varsigma_1)$ and $\c{G}_2 : (ţ_2, \varsigma_2)$ are PFGs such that $\c{G}_1 \propto \c{G}_2$ is complete, then both $\c{G}_1$ or $\c{G}_2$ must be complete.*

Proof. Case 1: If $\H{u}_1\H{u}_2 \in \breve{E}_1$, then $(\varsigma_1 \propto \varsigma_2)((\H{u}_1,\H{u}_2) = \varsigma_1(\H{u}_1\H{u}_2)$ and as $\c{G}_1 \propto \c{G}_2$ is complete,

$$\begin{aligned}\varsigma_1(\H{u}_1\H{u}_2) &= (\varsigma_1 \propto \varsigma_2)(\H{u}_1, \H{u}_2)\\ &= (ţ_1 \propto ţ_2)(\H{u}_1)(ţ_1 \propto ţ_2)(\H{u}_2)\\ &= ţ_1(\H{u}_1)ţ_1(\H{u}_2)\end{aligned}$$

Case2: If $\H{u}_1\H{u}_2 \in \breve{E}_2$, then this case is similar to case 1.

Combining the last Corollary and Theorem, we obtain the following result:

Corollary 2. *$\c{G}_1 : (ţ_1, \varsigma_1)$ and $\c{G}_2 : (ţ_2, \varsigma_2)$ are fuzzy complete PFGs if and only if $\c{G}_1 \propto \c{G}_2$ is a complete PFG.*

An interesting property of complement is given next.

Theorem 3. *If $Ģ_1 : (ţ_1, ς_1)$ and $Ģ_2 : (ţ_2, ς_2)$ are fuzzy complete graphs, then* $\overline{Ģ_1 \propto Ģ_2} \simeq \overline{Ģ_1} \propto \overline{Ģ_2}$.

Proof. Let Ģ: $(ţ, \overline{ς}) = \overline{Ģ_1 \propto Ģ_2}$, $\overline{ς} = \overline{ς_1 \propto ς_2}$, $\overline{Ģ^*} = (\H{U}, \overline{\breve{E}})$, $\overline{Ģ_1} : (ţ_1, \overline{ς_1})$, $\overline{Ģ_1^*} = (\H{U}_1, \overline{\breve{E}_1})$, $\overline{Ģ_2} : (ţ_2, \overline{ς_2})$, $\overline{Ģ_2^*} = (\H{U}_2, \overline{\breve{E}_2})$ and $\overline{Ģ_1} \propto \overline{Ģ_2} : (ţ_1 \propto ţ_2, \overline{ς_1} \propto \overline{ς_2})$. We only need to show $\overline{ς_1 \propto ς_2} = \overline{ς_1} \propto \overline{ς_2}$. For any arc e in Ĕ joining nodes of Ű,we have the following cases:

The cases $ű_1ű_2 \in \breve{E}_1$, then as $Ģ_1$ is complete, $\overline{ς_1}(e) = 0$. On the other hand $\overline{ς_1 \propto ς_2}(e) = 0$ since $ű_1ű_2 \notin \overline{\breve{E}_1}$.

The case $ű_1ű_2 \in \breve{E}_2$ is similar to the above case. The case $ű_1ű_2 \notin \breve{E}_1 \cup \breve{E}_2$ is not possible to occur as both $Ģ_1$ and $Ģ_2$ are complete.

In all cases $\overline{ς_1 \propto ς_2} = \overline{ς_1} \propto \overline{ς_2}$ and therefore, $\overline{Ģ_1 \propto Ģ_2} \simeq \overline{Ģ_1} \propto \overline{Ģ_2}$.

3. Unbiased Product Fuzzy Graphs

We begin this section by recalling the definition of unbiased (balanced) PFGs from [7] and then proving the following lemma that we shall use to give necessary and sufficient conditions for the corona product of two unbiased PFGs to be unbiased.

Definition 8. [7] *The density of a PFG is* $d(Ģ) = \dfrac{2 \sum\limits_{ű\ddot{y} \in \breve{E}} (ς(ű\ddot{y}))}{\sum\limits_{ű, \ddot{y} \in \H{U}} (ţ(ű) \wedge ţ(\ddot{y}))}$. *Ģ is unbiased (balanced) if $d(H) \leq d(Ģ)$ for any non-empty product fuzzy subgraphs H of Ģ.*

Lemma 4. *Let $Ģ_1$ and $Ģ_2$ be PFGs. Then $d(Ģ_i) \leq d(Ģ_1 \propto Ģ_2)$ for $i = 1, 2$ if and only if $d(Ģ_1) = d(Ģ_2) = d(Ģ_1 \propto Ģ_2)$.*

Proof. If $d(Ģ_i) \leq d(Ģ_1 \propto Ģ_2)$ for $i = 1, 2$, then

$$
\begin{aligned}
d(Ģ_1) &= 2\Big(\sum_{ű_1, ű_2 \in \H{U}_1} ς_1(ű_1ű_2)\Big)\Big/\Big(\sum_{ű_1, ű_2 \in \H{U}_1} (ţ_1(ű_1) \wedge ţ_1(ű_2))\Big) \\
&\geq 2\Big(\sum_{\substack{ű_1, ű_2 \in \H{U}_1 \\ \ddot{y}_1, \ddot{y}_2 \in \H{U}_2}} ς_1(ű_1ű_2)ţ_2(\ddot{y}_1)ţ_2(\ddot{y}_2)\Big)\Big/\Big(\sum_{\substack{ű_1, ű_2 \in \H{U}_1 \\ \ddot{y}_1, \ddot{y}_2 \in \H{U}_2}} (ţ_1(ű_1)ţ_1(ű_2)ţ_2(\ddot{y}_1)ţ_2(\ddot{y}_2))\Big) \\
&\geq 2\Big(\sum_{\substack{ű_1, ű_2 \in \H{U}_1 \\ \ddot{y}_1, \ddot{y}_2 \in \H{U}_2}} ς_1(ű_1ű_2)ς_2(\ddot{y}_1\ddot{y}_2)\Big)\Big/\Big(\sum_{\substack{ű_1, ű_2 \in \H{U}_1 \\ \ddot{y}_1, \ddot{y}_2 \in \H{U}_2}} (ţ_1(ű_1)ţ_1(ű_2)ţ_2(\ddot{y}_1)ţ_2(\ddot{y}_2))\Big) \\
&\geq 2\Big(\sum_{\substack{ű_1, ű_2 \in \H{U}_1 \\ \ddot{y}_1, \ddot{y}_2 \in \H{U}_2}} ς_1 \propto ς_2((ű_1\ddot{y}_1)(ű_2\ddot{y}_2))\Big)\Big/\Big(\sum_{\substack{ű_1, ű_2 \in \H{U}_1 \\ \ddot{y}_1, \ddot{y}_2 \in \H{U}_2}} (ţ_1 \propto ţ_2((ű_1, \ddot{y}_1)(ű_2, \ddot{y}_2))\Big)
\end{aligned}
$$

$$= \quad d(Ģ_1 \propto Ģ_2).$$

Hence in all cases $d(Ģ_1) \geq d(Ģ_1 \propto Ģ_2)$ and thus $d(Ģ_1) = d(Ģ_1 \propto Ģ_2)$. Similarly, $d(Ģ_2) = d(Ģ_1 \propto Ģ_2)$. Therefore, $d(Ģ_1) = d(Ģ_2) = d(Ģ_1 \propto Ģ_2)$. The converse is trivial.

Theorem 4. *Let $Ģ_1$ and $Ģ_2$ be unbiased PFGs. Then $Ģ_1 \propto Ģ_2$ is unbiased if and only if $d(Ģ_1) = d(Ģ_2) = d(Ģ_1 \propto Ģ_2)$.*

Proof. If $Ģ_1 \propto Ģ_2$ is unbiased, then $d(Ģ_i) \leq d(Ģ_1 \propto Ģ_2)$ for $i = 1, 2$ and by Lemma 4, $d(Ģ_1) = d(Ģ_2) = d(Ģ_1 \propto Ģ_2)$.

Conversely, if $d(Ģ_1) = d(Ģ_2) = d(Ģ_1 \propto Ģ_2)$ and H is a product fuzzy subgraph of $Ģ_1 \propto Ģ_2$, then there exist product fuzzy subgraphs H_1 of $Ģ_1$ and H_2 of $Ģ_2$. As $Ģ_1$ and $Ģ_2$ are unbiased and $d(Ģ_1) = d(Ģ_2) = m_1/k_1$, then $d(H_1) = a_1/b_1 \leq m_1/k_1$ and $d(H_2) = a_2/b_2 \leq m_1/k_1$. Thus $a_1k_1 + a_2k_1 \leq b_1m_1 + b_2m_1$ and hence $d(H) \leq (a_1 + a_2)/(b_1 + b_2) \leq m_1/k_1 = d(Ģ_1 \propto Ģ_2)$. Therefore, $Ģ_1 \propto Ģ_2$ is unbiased.

We end this section with the following result which states that unbiased notion is preserved under isomorphism:

Theorem 5. *Let $Ģ_1$ and $Ģ_2$ be isomorphic PFGs. If one of them is unbiased, then the other is unbiased.*

Proof. Suppose $Ģ_2$ is unbiased and let $h : \H{Ű}_1 \to \H{Ű}_2$ be a bijection such that $ţ_1(ű) = ţ_2(h(ű))$ and $ς_1(ű\ddot{y}) = ς_2(h(ű)h(\ddot{y}))$ for all $ű, \ddot{y} \in \H{Ű}_1$. Now $\sum_{ű \in \H{Ű}_1} ţ_1(ű) = \sum_{ű \in \H{Ű}_2} ţ_2(ű)$ and $\sum_{ű\ddot{y} \in \breve{E}_1} ς_1(ű\ddot{y}) = \sum_{ű\ddot{y} \in \breve{E}_2} ς_2(ű\ddot{y})$. If $H_1 = (ţ_1, ς_1)$ is a product fuzzy subgraph of $Ģ_1$ with underlying set W, then $H_2 = (ţ_2, ς_2)$ is a product fuzzy subgraph of $Ģ_2$ with underlying set $h(W)$ where $ţ_2(h(ű)) = ţ_1(ű)$ and $ς_2(h(ű)h(\ddot{y})) = ς_1(ű\ddot{y})$ for all $ű, \ddot{y} \in W$. Since $Ģ_2$ is unbiased, $d(H_1) \leq d(Ģ_2)$ and so $2\frac{\sum_{ű\ddot{y} \in \breve{E}_1} ς_2(h(ű)h(\ddot{y}))}{\sum_{ű,\ddot{y} \in \H{Ű}_1}(ţ_2(ű) \wedge ţ_2(\ddot{y}))} \leq 2\frac{\sum_{ű\ddot{y} \in \breve{E}_1} ς_2(ű\ddot{y})}{\sum_{ű,\ddot{y} \in \H{Ű}_1}(ţ_2(ű) \wedge ţ_2(\ddot{y}))}$. Hence

$$2\frac{\sum_{ű\ddot{y} \in \breve{E}_1} ς_1(ű\ddot{y})}{\sum_{ű,\ddot{y} \in \H{Ű}_1}(ţ_2(ű) \wedge ţ_2(\ddot{y}))} \leq 2\frac{\sum_{ű\ddot{y} \in \breve{E}_1} ς_1(ű\ddot{y})}{\sum_{ű,\ddot{y} \in \H{Ű}_1}(ţ_2(ű) \wedge ţ_2(\ddot{y}))}.$$

Therefore, $Ģ_1$ is unbiased.

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